

Oscillatory Universe, dark energy equation of state and general relativity

P. P. Ghosh^{1*}, Saibal Ray^{2†}, A. A. Usmani^{3‡}, Utpal Mukhopadhyay^{4§}

¹*Department of Physics, A. J. C. Bose Polytechnic, Berachampa, North 24 Parganas, Devalaya 743424, West Bengal, India*

²*Department of Physics, Government College of Engineering & Ceramic Technology, Kolkata 700010, West Bengal, India*

³*Department of Physics, Aligarh Muslim University, Aligarh 202002, Uttar Pradesh, India*

⁴*Department of Mathematics, Satyabharati Vidyapith, Nabapalli, North 24 Parganas, Kolkata 700126, West Bengal, India*

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ABSTRACT

The concept of oscillatory Universe appears to be realistic and buried in the dynamic dark energy equation of state. We explore its evolutionary history under the frame work of general relativity. We observe that oscillations do not go unnoticed with such an equation of state and that their effects persist later on in cosmic evolution. The ‘classical’ general relativity seems to retain the past history of oscillatory Universe in the form of increasing scale factor as the classical thermodynamics retains this history in the form of increasing cosmological entropy.

Key words: gravitation - cosmological parameters - cosmology: theory - early Universe.

1 INTRODUCTION

Although, the belief in oscillatory Universe dates back to the ancient times ((Kanekar, Sahni & Shtanov 2001) and references cited therein), a scientific model for it could only be proposed during the first half of the twentieth century (Friedman 1922). At a stage, oscillatory Universe was considered as one of the main possibilities of cosmic evolution (Dicke 1965). However, Durrer & Laukenmann (1996) showed it as a viable alternative to inflation. Throughout the past century, the idea has been of scientific importance rather than of mere belief, which has been a focus of many theoretical investigations till date as summarized below.

The building up of cosmological entropy in each cycle of oscillation as shown by Tolman (1934) is fated to a thermodynamical end. This had suggested that the Universe had a beginning at a finite time, and thus had undergone a finite number of cycles (Zeldovich 1967), opposed to the idea of steady-state Universe. When treated classically, it reaches a point of singularity at the end of every cycle resulting in a total breakdown for the general relativity (GR). However, there are models with contracting epoch preceding a bounce in M-theories (Khoury et al. 2002; Steinhardt & Turok 2002), braneworlds (Kanekar, Sahni & Shtanov 2001; Shtanov & Sahni 2003; Hovdebo & Myers 2003; Foffa 2003; Burgess et al. 2004) and loop quantum cosmologies

(Lidsey et al. 2004; Ashtekar, Pawłowski & Singh 2006). The quantum effects built into these studies aim to provide a non singular framework for the bounces that avoid a kind of singularity hit by GR. Besides, they all admit a cosmological evolution that has undergone an early oscillatory phase for a finite time, and has finally led to inflation as we see it now. Evolution of the expanding Universe has also been studied using scalar, vector and tensor cosmological perturbation theories (Mukhanov, Feldman & Brandenberger 1992; Noh & Hwang 1982; Bartolo et al. 2004; Nakamura 2007). Scalar field models that involve contracting phases are proposed as alternatives to the inflationary scenario. These models have also been explored to distinguish such phases from purely expanding cosmologies. Vector perturbations uptill the second order have recently been considered as the scalar perturbations may not be able to distinguish it at first order (Mena, Mulryne & Tavakol 2007). Oscillatory model arising from linearized R^2 theory of gravity (Corda 2008) has also been studied, which is found in reasonable agreement with some observational results like the cosmological red shift and the Hubble law. Saha & Boyadjiev (2003) obtained the oscillatory mode of expansion of the Universe from a self-consistent system of spinor, scalar and gravitational fields in presence of a perfect fluid and cosmological term Λ (Zeldovich 1967; Weinberg 1989).

Various observations (Riess et al. 2004; Spergel et al. 2003; Tegmark et al. 2004; Perlmutter et al. 1999) suggest that our Universe is accelerating. And the reason behind it is believed to be a mysterious *dark energy*, a term coined

* E-mail: parthapapai@gmail.com

† E-mail: saibal@iucaa.ernet.in

‡ E-mail: anisul@iucaa.ernet.in

§ E-mail: umsbv@yahoo.in

recently in 1999 but its history traces back as late as Newton's time (Calder & Lahav 2008). The cosmological constant Λ , first adopted and then abandoned by Einstein for his static model of the Universe (Einstein 1917, 1918, 1931), is considered as one of the candidates for the dark energy. The other candidate is a scalar field often referred to as quintessence (Watterich 1988; Ratra & Peebles 1988; Caldwell, Dave & Steinhardt 1998). It is, however, speculated that Λ is a dynamical term rather than a constant.

The dark energy may be described as a perfect fluid through the equation of state (EOS) relating the fluid pressure and matter density of the physical system through the relation $p = \omega\rho$, where $\omega(t) \equiv \omega$ is the barotropic dark energy equation of state parameter. It plays a significant role in the cosmological evolution. This is in general a function of time and may be a function of scale factor or redshift (Chervon & Zhuravlev 2000; 2001 1982; Peebles & Ratra 2003). Recently, Usmani et al. (2008) have proposed a time dependent $\omega(t)$ for the study of cosmic evolution with the equation of state parameter

$$\omega(t) = \omega_0 + \omega_1(t\dot{H}/H). \quad (1)$$

Here, $H \equiv H(t)$ is the time varying Hubble parameter. The dynamics of both expanding and contracting epoch of the Universe is buried in the sign of \dot{H} in this simple expression. The $\dot{H}=0$ represents a linearly expanding Universe with $\omega(t) = \omega_0$.

A close system like an oscillatory Universe may be created without violating any known conservation law of Physics as its total mass (energy) is zero, which is indeed possible (Tryon 1973). A Universe may enter into an epoch of expansion followed by an epoch of contraction after every bounce. For an expanding Universe that was created at $t = 0$ with $H(t) = 0$, we may arrive at two situations (i) linearly expanding Universe represented by $H = t\dot{H}$ (ii) and Universe that has undergone an adiabatic expansion to a finite H_i followed by a linear expansion (for example, our Universe) that may be represented by another condition $H \gg t\dot{H}$ provided $H_i \gg t\dot{H}$. Having the same theoretical framework these two cases may be represented by a single set of equations (Usmani et al. 2008) with a difference that inflation leads to a scaling in ω . Thus, it is the equation of state that distinguishes between the two. After achieving its maximum radius at time $t = t_0$ ($\tau = 0$) with $\dot{H} = 0$ and $H(t_0) = H_{max}$, the Universe starts collapsing due to its own mass and enters into an epoch of contraction with a negative sign of \dot{H} . Thus at a later time $t = t_0 + \tau$, H is defined as $H = H_{max} - \tau\dot{H}$, which would continue to decrease till Universe achieves its minimum radius. At preceding times closer to it, a 'condition of oscillation', $H(t) \ll |t\dot{H}|$, would hold. This condition would still at a time when Universe sets to expand again.

The above equation of state enables us to handle this third possibility at least qualitatively when employed in the framework of general relativity. The history of oscillations does not go unnoticed by the classical thermodynamics since cosmological entropy (unidirectional thermodynamical scale of time) keeps on growing after every cycle (Tolman 1934). The generalized second law (GSL) of thermodynamics relates the dark energy equation of state with cosmological entropy (Bakenstein 1973; Gibbons & Hawking 1977a,b; Unruh & Wald 1982; Davies 1988). Thus the equation of

state, which translates itself into entropy, would manifest similar properties despite the fact that relativity breaks down at singularities, whose details are not required in the framework. An observer outside the Universe, who measures a growing time (or growing entropy) from point of creation, would distinguish a state in an oscillation with an identical state in the next oscillation with a decrease in \dot{H} . This would result in different solutions of field equations for identical states in two different oscillations. The state of expansion, contraction and oscillation of the Universe is buried in \dot{H} . We find no reason to believe, why field equations of general relativity would not yield a valid solution for any value of \dot{H} no matter how many times the Universe has hit the singularity in the middle and has come up to a finite size again and again for 'unknown' reasons.

2 FIELD EQUATIONS AND THEIR SOLUTIONS

The Einstein field equations of general relativity are

$$R^{ij} - Rg^{ij}/2 = -8\pi G [T^{ij} - (\Lambda/8\pi G)g^{ij}], \quad (2)$$

where Λ is time-dependent Cosmological term with vacuum velocity of light being unity in relativistic units. For the spherically symmetric Friedmann-Lemaître-Robertson-Walker metric, Einstein field equations (2) yield Friedmann and Raychaudhuri equations, respectively,

$$3H^2 + 3k/a^2 = 8\pi G\rho + \Lambda, \quad (3)$$

$$3H^2 + 3\dot{H} = -4\pi G(\rho + 3p) + \Lambda. \quad (4)$$

Here, $a = a(t)$ is the cosmic scale factor, k is the curvature constant, $H = \dot{a}/a$ is the Hubble parameter and G , ρ , p are the gravitational constant, matter-energy density and fluid pressure, respectively. The G is taken to be a constant quantity along with a variable Λ in which case the generalized energy conservation law may be derived (Shapiro, Solà & Štefančić 2005). Using equation of state ($p = \omega\rho$) in Eq. (4) and differentiating Eq. (3) with time t for flat Universe ($k = 0$), we arrive at

$$-4\pi G\rho = \dot{H}/(1 + \omega). \quad (5)$$

In the light of the studies by Ray, Mukhopadhyay & Duttachowdhury (2007), we use the *ansatz*, $\Lambda = AH^3$. As argued by Usmani et al. (2008) this *ansatz* may find realization in the framework of self consistent inflation model (Dymnikova & Khlopov 2000) in which time-dependent Λ is determined by the rate of Bose condensate evaporation with $A \sim (m_B/m_P)^2$ (where m_B is the mass of bosons and m_P is the Planck mass).

The equation of state along with Eqs. (4), (5) and above *ansatz* leads to

$$d\dot{H}/dH + 3(1 + \omega)H = (1 + \omega)AH^3/2\dot{H}. \quad (6)$$

The condition $H(t) \ll |t\dot{H}|$ with $\pm\dot{H}$ simplifies it further

$$\pm d\dot{H}/dH + 3\omega_1\tau\dot{H} = 0 \quad \text{or} \quad d\dot{H}/dH \pm 3\omega_1\tau\dot{H} = 0, \quad (7)$$

A change of sign for \dot{H} would amount to the change of sign for ω_1 . Thus, we have same solution set for $\pm\dot{H}$ with $\pm\omega_1$ as

$$a(t) = C \exp(B[\ln(T) - 1]), \quad (8)$$

$$H(t) = \frac{B}{t} \ln(T), \quad (9)$$

$$\omega(t) = \omega_0 + \omega_1 \left(\frac{1}{\ln(T)} \right), \quad (10)$$

$$\rho(t) = - \left(\frac{1}{4\pi G} \right) \left[\frac{B}{t^2(1 + \omega(t))} \right], \quad (11)$$

$$p(t) = \omega(t)\rho(t), \quad (12)$$

$$\Lambda(t) = \frac{AB^3}{4t^2} [(\ln(T))^3 - 3(\ln(T))^2 + 6\ln(T) - 6], \quad (13)$$

where, $B = t/E\tau$, $T = DE\tau t$ and C and D are integration constants.

3 DISCUSSION AND CONCLUSION

The logarithm $\ln(T)$ demands a positive definite value for T . Thus E , D and ω_1 must have identical sign. Therefore, second term of Eq. (10) i.e. $\omega_1/\ln(T)$ is positive for $T > 1$ and negative in the range $0 < T < 1$. We find $\omega_1 = 0$ for a linearly expanding Universe and a range $-2/3 < \omega_1 < -0.46$ for an inflationary Universe (Usmani et al. 2008). Thus, it is always negative. For a contracting Universe \dot{H} is negative, which makes the second term of Eq. (1) positive. This, when compared with Eq. (10), suggests that ω_1 is positive for $T > 1$ and negative for $T < 1$. Thus, for $D = 1$, we obtain a condition, $E = 3\omega_1 > 1/\tau t$. Here $t \rightarrow \inf$ means $\omega_1 \rightarrow 0$, which represents a linearly expanding Universe with $\omega(t) = \omega_0 = -1/3$. For an observer sitting outside the Universe, the rate \dot{H} would appear to be smaller and smaller for the increasing number of oscillations as t would appear to be larger and larger. It is obvious from Eq. (9) that H is positive definite that means it grows from zero to a maximum value during the epoch of expansion and then approaches to zero during the epoch of contraction. We also observe that Eq. (11) for the density is singular at $1 + \omega(t) = 0$, so is the pressure ($p = \omega\rho$), which turns out to be positive for $0 > \omega(t) > -1$. The same restriction is found for the expanding Universe using same equation of state (Usmani et al. 2008), which is in agreement with the results of GSL of thermodynamics that too restricts the equation of state, $\omega(t) > -1$, in an expanding Universe. Beyond this limit, for example $\omega(t) > 0$, pressure is negative.

Let us now try to extract the two phases of expanding and contracting Universe. As mentioned earlier, we had the condition $-1 < w < -0.46$ (Usmani et al. 2008) which we must satisfy here also and thereby take $w_0 = -1/3$ and $-2/3 < w_1 < -0.46$. With this our ρ will always be positive either Universe is expanding or contracting. In our solution T depends upon the constants D , E , τ and also t . E , via the relation $E = 3w_1$, is a negative quantity. Thus D must be negative to make T positive and between 0 and 1. Hence

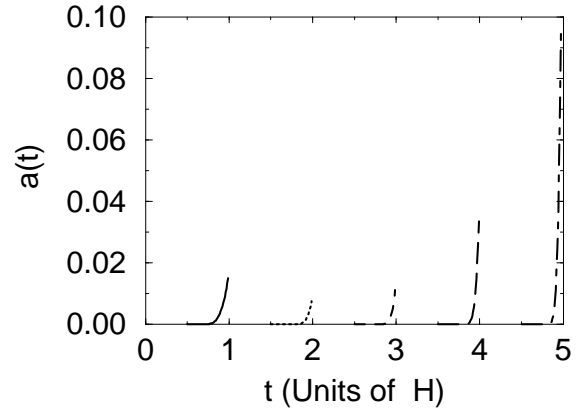


Figure 1. Growth of scale factor $a(t)$ for the successive oscillations having equal epoch of expansion and contraction. The solid, dotted, dashed, long-dashed and chain curves represent $a(t)$ during first, second, third, fourth, fifth and sixth oscillations, respectively. The oscillation period for the Universe is assumed to be ($t = 1$ units of H).

pressure will be positive and Universe will contract. It also depends upon constant D , so if $|D|$ multiplied by τ is less than 1 pressure would be positive. In other words if E is very very small, even for a large τ , pressure will be positive and Universe will contract. But with increasing τ , $|D|\tau$ will eventually be greater than equal to one and pressure will again be negative and the Universe will start expanding.

We, therefore, construct the story for a Universe like ours, which had started with an adiabatic expansion with $\omega(t) < -1/3$, then it had been slowed down to a linearly expanding Universe with $\omega(t) = -1/3$, which was then followed by an epoch of contraction with a positive value of $\omega_1/\ln(T)$. For smaller values of T or t , $\omega_1/\ln(T)$ may reach the condition $\omega(t) > 0$, which will make the pressure positive and thus would receive a bounce. This process would continue repeating without violating any known conservation law. However, $\omega_1/\ln(T) \rightarrow 0$ for $T \rightarrow \inf$. Thus, the Universe would appear to be a linearly expanding Universe with $\omega(t) = \omega_0 = -1/3$ and would never reach the condition for contraction or bounce i.e. $\omega(t) > 0$. Thus, it recommends for initial oscillations only. One may suggest improvements in the equation of state, however we have shown a way to incorporate oscillatory Universe in general relativity through it.

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